【Chapter 3 **Wave Properties of Particles**】

.De Broglie waves

A moving body behaves in certain ways as though it has a wave nature.

* for photon

$$P = h\nu/c = h/\lambda$$

Photon wavelength $\longrightarrow$ $\lambda = h/P$………(3.1)

De Broglie Suggested (3.1) is general one that applies to **material particles** as well as to photons.

$\longrightarrow$ De Broglie wavelength

$$\lambda = h/P = h/mv$$

$$(m = \frac{m_o}{\sqrt{1 - v^2/c^2}})$$

Example 3.1

Find the de Brogli wavelengths of

(a) 46-g golf ball with a v = 30 m/s

(b) e' with a v = 10^7m/s

(1) v<<c $\longrightarrow$ m =m_o

λ= h/mv =6.63x10^{-34}Js/(0.046kg)(30m/s)=4.8x10^{-34}m

wavelength is very small

λ= h/mv =6.63x10^{-34}Js/(9.1x10^{-31}kg)(10^7m/s)=7.3x10^{-11}m

=0.73Å

the radius of H atom = 5.3x10^{-11}m=0.53 Å

wave character of moving e' is the key to understand atomic structure behavior

【3.2 Waves of probability】

Water wave ⟶ (varing quantity) height of water surface

Light wave ⟶ E& H fields

How about matter waves

⟶ Wave function Ψ

The value of wave function associated with a moving body at the particular point x , y, z at time t is related to the likehood of finding the body there at the time.

*Ψ has no direct physical significance

$0 \le$ probability ≤ 1

but the amplitude of wave am be positive or negative

⟶ no negative probability

⟹ $|\varphi|^2$:squae of the absolute value of wave function

⟶ probability density

** The probability of experimentally finding the body described by the wave functionΨat the point x , y , z at time t is proportional to $|\varphi|^2$ there at t.

wave functionΨthat described a particle is spread out is spall, but it does not mean that the particle itself is spread out.

【3.3 Describing a wave】

de Broglie wave velocity v_p

$v_p = \nu\lambda (\lambda = h/mv)$

$h\nu = mc^2 \longrightarrow \nu = mc^2/h$

De Broglie phase velocity $v_p = \nu\lambda = (mc^2/h)(h/mv) = c^2/v$ (v = particle velocity)

Because V<C

$\longrightarrow$ de Broglie waves always travel faster than light !!

$\longrightarrow$ Phase velocity, group velocity.

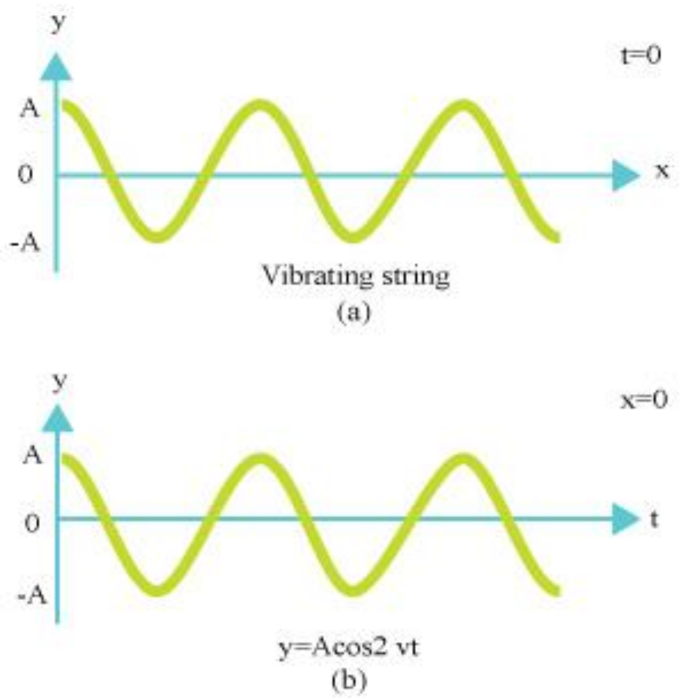


Figure3.1 (a) The appearance of a wave in a stretched string at a certain time.
(b) How the displacement of a point on the string varies with time.

At $x=0$, $y=A\cos(2\pi\nu t)$ for time=t

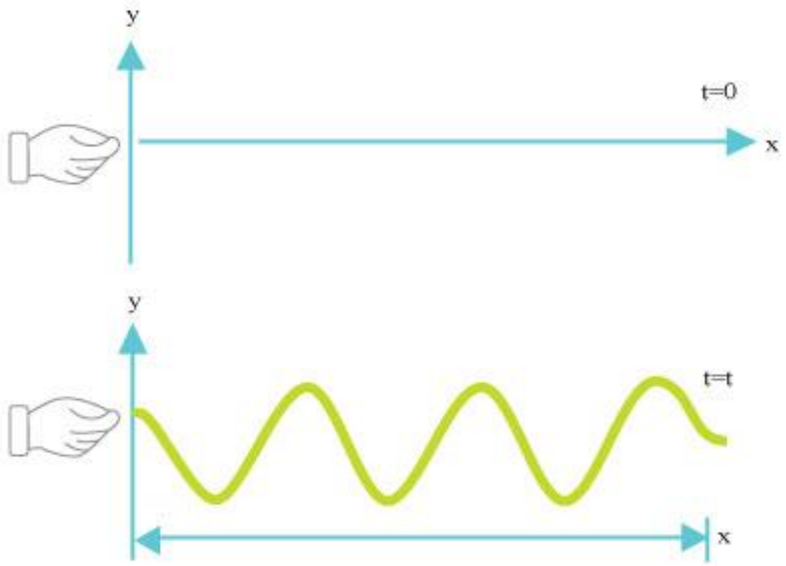


Figure3.2   Wave propagation.

$x = v_p t$, $t = x/v_p$

$y=A\cos 2\pi\nu(t-x/v_p)$

the amplitude for $y(x,t) = y(0,t-x/v_p)$

$y = A\cos 2\pi(\nu t - \frac{\nu_x}{v_p})$ $v_p = \nu\lambda$

$\longrightarrow$ $y = A\cos 2\pi(\nu t - x/\lambda)$

angular freguency $\omega= 2\pi\nu$ wave number $k= 2\pi/\lambda=\omega/v_p$

$\longrightarrow$ $y = A\cos(\omega t - kx)$

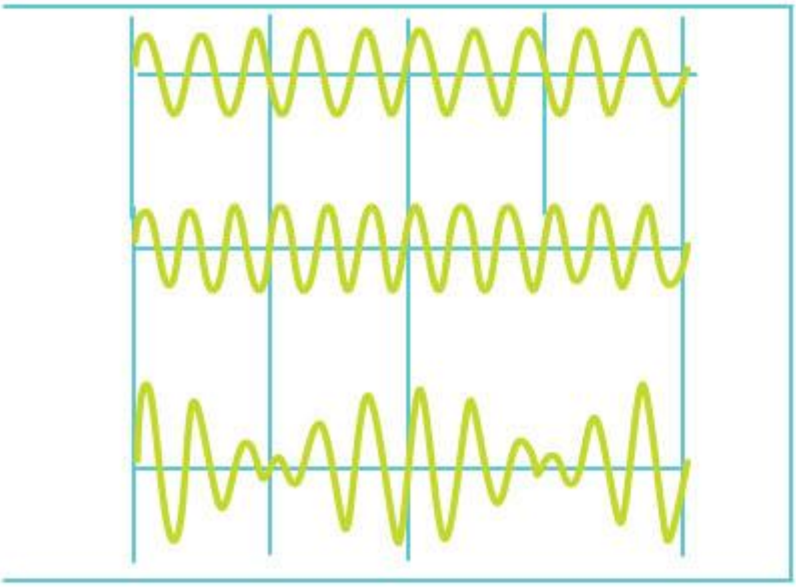


Figure3.3   A wave group.

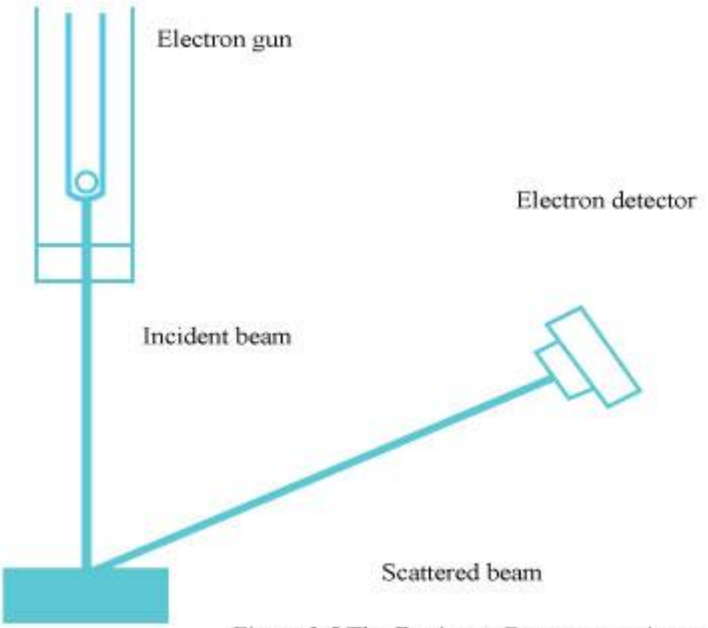


Figure 3.5 The Davisson-Germer experiment

Figure3.4   Beats are produced by the superposition of two waves with different frequencies.

The amplitude of de Broglie waves → probability

De Broglie wave can not be represented by $y=A\cos(wt-kx)$

. wave representation of a moving body → wave packet

wave group

. An example is a beat. (two sound waves of the same amplitude but slightly different frequencies)

original 440, 442 Hz → hear fluctuating sound of 441 Hz with 2 beats/s

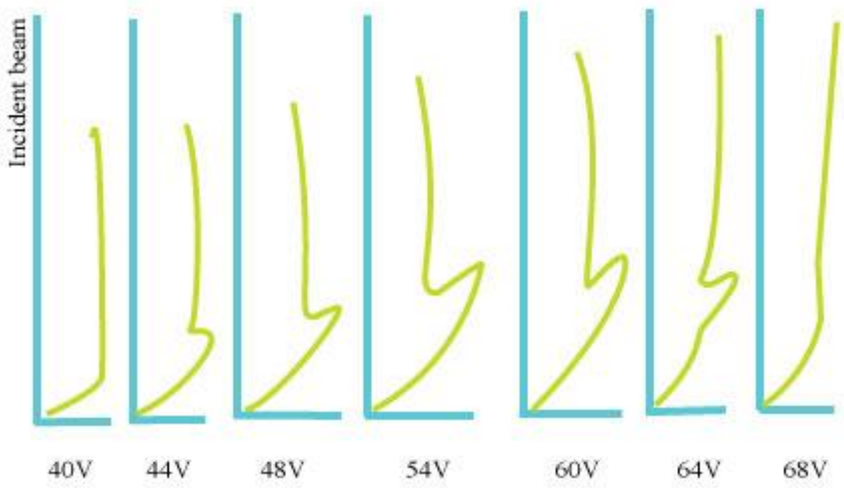


Figure3.5 The Davisson-Germer experiment.

a wave group: superposition of individual waves of different λ which interference with one another

→ variation in amplitude → define the group shape

(1) If the velocities of the waves are the same $\longrightarrow$ the velocity of wave group is common phase velocity

(1)If the phase velocity varies with λ

$\longrightarrow$ an effect called dispertion

$\longrightarrow$ individual waves do not proceed together

$\longrightarrow$ wave group has a velocity different from the phase velocities

$\longrightarrow$ the case of de Broglie wave

- **group velocity**

$y_1 = A\cos[(\omega t - ks)]$

$y_2 = A\cos[(\omega+\Delta\omega)t - (k+\Delta k)x]$

$\longrightarrow$ $y = y_1+y_2$

$=2A\cos 1/2[(2\omega+\Delta\omega)t - (2k+\Delta k)x]\cos 1/2(\Delta\omega t-\Delta kx)$

because $\Delta\omega \ll \omega \longrightarrow 2\omega+\Delta\omega \approx 2\omega$

$\Delta k \ll k \longrightarrow 2k+\Delta k \approx 2k$

$\longrightarrow$ $Y = 2A\cos(\omega t - kx)\cos[(\Delta\omega/2)t - (\Delta k/2)x]$

A wave of angular frequency ω & wave number k that has superimposed upon it a modulation of angular frequency $1/2\Delta\omega$

& of wave number $1/2\Delta k$

Modulation produce wave group

v_p = $\frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda} = \nu\lambda$ phase velocity

$v_g = \Delta\omega/\Delta k = d\omega/dk$ group velocity

for de Broglie waves

$$\varpi = 2\pi\nu = \frac{2\pi\nu mc^2}{h} = = \frac{2\pi m_o c^2}{h\sqrt{1-v^2/c^2}}$$ (because $h\nu=mc^2$)

$$k = \frac{2\pi}{\lambda} = \frac{2\pi m v}{h} = \frac{2\pi m_o v}{h\sqrt{1-v^2/c^2}}$$ (because $\lambda=h/mv$)

* both ω&k are functions of body's v $v_g = d\omega/dk = \frac{d\omega/dv}{dk/dv}$

$$\frac{d\omega}{dv} = \frac{2\pi m_o v}{h\left(1-v^2/c^2\right)^{\frac{2}{3}}} \quad , \quad \frac{dk}{dv} = \frac{2\pi m_o}{h\left(1-v^2/c^2\right)^{\frac{3}{2}}}$$

$\longrightarrow$ $v_g = v$ (de Broglie group velocity)

De Broglie wave group associated with a moving body travels with the same velocity as the body.

De Broglie phase velocity $v_p = \omega/k = c^2/v$

v_p > velocity of the body v > c

($\because$ it is not the motion of the body)

Ex 3.3 :

An e' has a de Broglie wavelength of 2pm=$2x10^{-12}$m.Find its kinetic energy & the phase & group velocity of its de Broglie waves.

(a) E =E_o+kE $\longrightarrow$ kE =E – E_o=$\sqrt{E_o^{\ 2} + p^2c^2} - E_o$

pc = hc/λ= $(4.136x10^{-15}$ev.s)$(3x10^8$m/s)/$(2x10^{-12})$ =

$6.2x10^5$ev=620kv

the rest energy of e' is E_o=511kv

$\longrightarrow$ kE=$\sqrt{(511)^2 + (620)^2} - 511 = 292$ kev

(b) e' velocity

$$E = \frac{E_o}{\sqrt{1 - v^2 / c^2}} \longrightarrow v = c\sqrt{1 - E_o^{\ 2} / E^2} = 0.771c$$

$\therefore$ $v_p = c^2/v = 1.3c$, $v_g = v = 0.771c$

3.5 particle diffraction $\longrightarrow$ e'-beam diffraction

$\longrightarrow$ confirm de Broglie waves

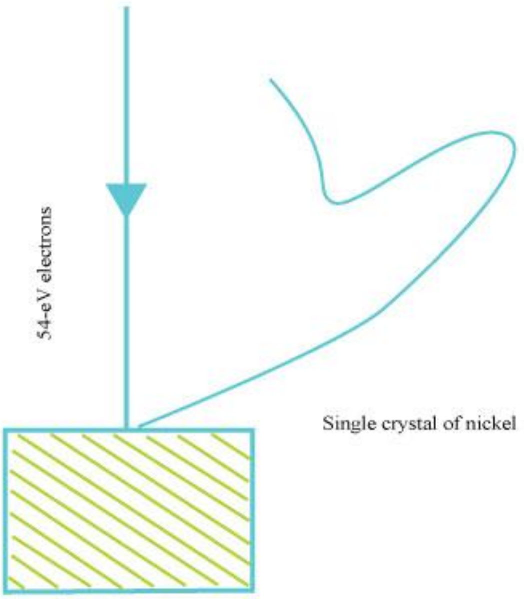


Figure3.6 Results of the Davisson-Germer experiment.

The method of plotting is such that the intensity at any angle is propotional to the distance of the curve at the angle from the point of scattering.

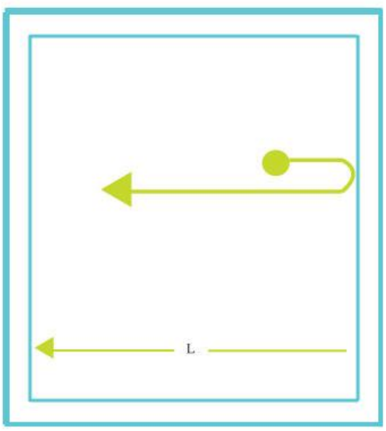


Figure3.7 The diffraction of the de Broglie waves by the target responsible for the results of Davisson and Germer.

$n\lambda=2d\sin\theta \longrightarrow \lambda=2d\sin\theta=0.165\text{nm} \quad \lambda=h/mv=0.166\text{nm}$

Figure3.8 Because the wavelengths of the fast electrons in an electron microscope are shorter than those of the light waves in an optical microscope can produce sharp images at higher magnifications. The electron beam in an electron microscope is focused by magnetic fields.

Figure3.9 A particle confined to a box of width L.

【3.6 particle in a box】

a prticle trapped in a box = a standing wave.

Ψmust be zero at the walls

⟶ $\lambda_n = 2L/n \quad n=1,2,3\ldots\ldots$

De Broglie wavelength of trapped particles.

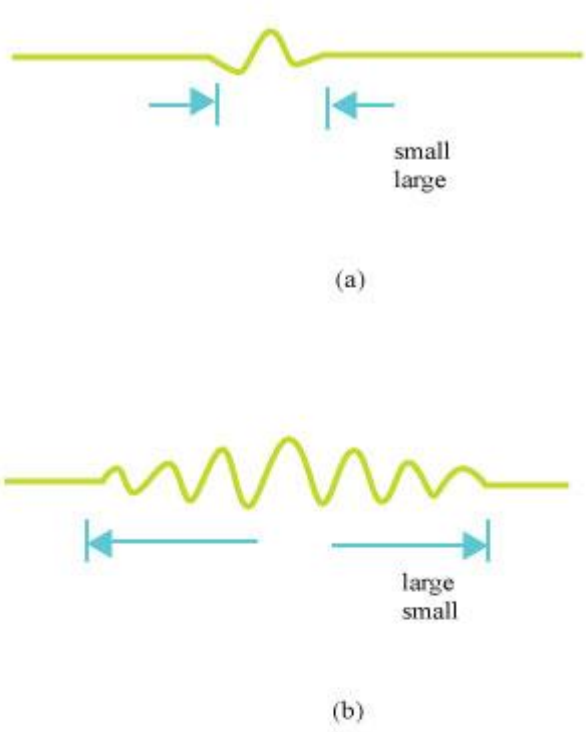


Figure3.10 Wave functions of a particle trapped in a box L wide.

$KE=1/2(mv^2)=(mv)^2/2m=h^2/(\lambda^2 2m)$

$\because \lambda_n=2L/n \rightarrow KE+v=E_n$ the energy for the particle in a box

$$\mathbf{E_n=n^2h^2/8mL^2} \rightarrow n=1,2,3........$$

Each permitted energy is called an

energy level.(n=quantum number)

This can be applied to any particle confined to a certain region of space.

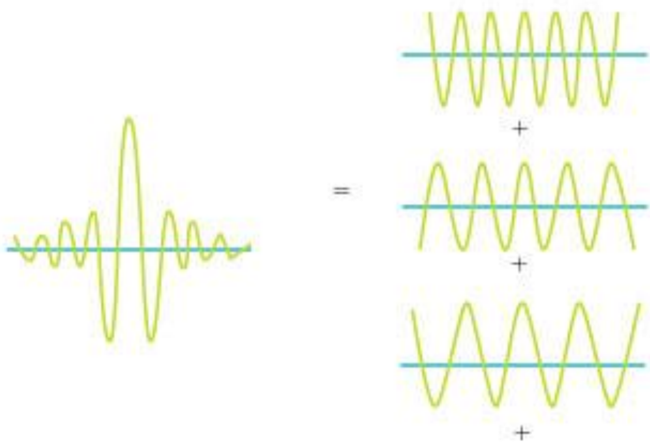
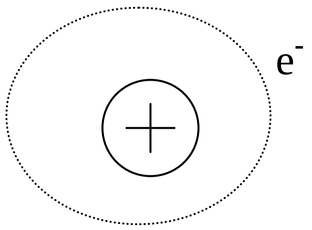


Figure3.11 Energy levels of an electron confined to a box 0.1nm wide.

$\Longrightarrow$ For example



1. Atraped particle cannot have an arbitrary energy, as a free particle can .

Confinement leads to restriction on its wave function that alloy the particle to have certain energies.

2. A trapped particle cannot have zero energy.

$\because$ de Broglie wavelength $\lambda = h/mv$ If $v = 0 \longrightarrow \lambda = \infty$

$\longrightarrow$ it can not be a trapped particle.

3. $\because$ $h = 6.63 \times 10^{-34}$Js very small

$\therefore$ only if m & L are very small, or we are not aware of energy quantization in our own experience.

Ex 3.4

An e' is in a box 0.1nm across, which is the order of magnitude of atomic distance, find its permitted energy.

$m = 9.1 \times 10^{-31}$kg & $L = 0.1\text{nm} = 10^{-10}$m

$E_n = n^2(6.63x10^{-34})/8x(9.1x10^{-31})(10^{-10})^2 = 6x10^{-18}n^2J = 38n^2ev$

When n=1 $\longrightarrow$ 38 ev

n=2 $\longrightarrow$ 152ev see fig 3.11

n=3 $\longrightarrow$ 342 ev

Ex 3.5

A long marble is in a box 10 cm across, find its permitted energies

$E_n = 5.5x10^{-64}n^2J$ n=1 $E=5.5x10^{-64}J$ $\longrightarrow$ $v=3.3x10^{-31}$ m/s

Which can not be experimentally distinguished from a stationary marble.

For a reasonable speed 1/3 m/s $\longrightarrow$ $n=10^{30}$!!

Energy levels are very close $\longrightarrow$ quantum effects are imperceptible

- Uncertainty principle

  * wave group narrower ⟶ particles position precise.

  However, λof waves in a narrow packet is not well defined ∵λ=h/mv ∴ P is not precise

  * A wide wave group ⟶ clearly definedλ but position is not certain

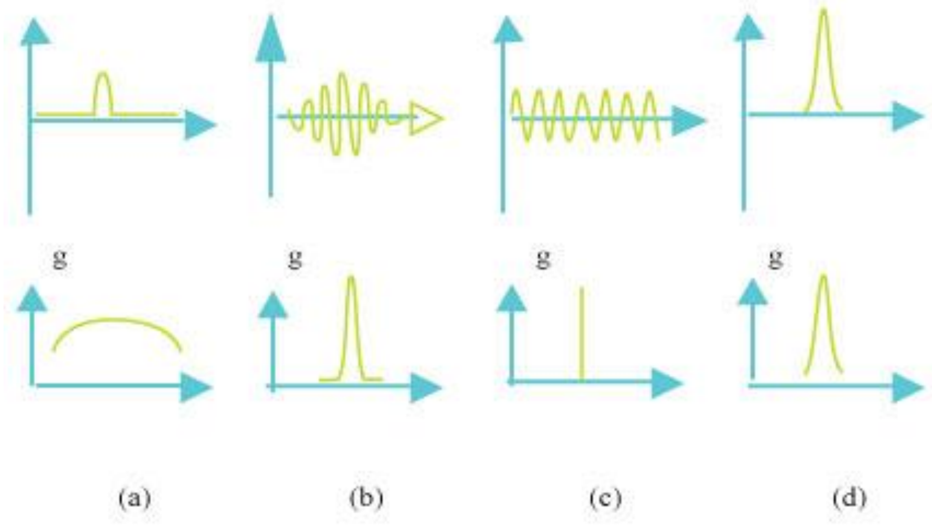


Figure3.12 (a) A narrow de Broglie wave group. The position of the particle can be precisely determined, but the wavelength (and hence the particle's momentum) cannot be established because there are not enough waves to measure accurately. (b) A wide wave group. Now the wavelength can be precisely determined but not the position of the particle.

uncertainty principle:

It is impossible to know both the exact position & exact momentum of an object at the same time.

Figure3.13 An isolated wave group is the result of superposing an infinite number of waves with different wavelengths. The narrower the wave group, the greater the range of wavelengths involves. A narrow de Broglie wave group thus means a well-defined position (Δx smaller) but a poorly defined wavelength and a large uncertainty Δp in the momentum of the particle the group represents. A wide wave group means a more precise momentum but a less precise position.

An infinite # of wave trains with different frequencies wave numbers and amplitude is required for an isolated group of arbitrary shape.

$\varphi(x) = \int_0^\infty g(k)\cos kx dk$ Fourier integral

g(k): amplitude of the waves varying with k , furrier transform of φ(x)

Figure3.14 The wave functions and Fourier transforms for (a) a pulse, (b) a wave group, (c) an wave train, and (d) a gaussian distribution. A brief disturbance needs a broader range of frequencies to describe it than a disturbance of greater duration. The Fourier transform of a gaussian function is also a gaussian function.

* wave numbers needed to represent a wave group extend from k=0 to k=∞, but for a group which length Δx is finite ⟶ waves which amplitudes g(k) are appreciable have wave number that lie within a finite interval Δk the shorter the group, the broader the range of wave numbers needed.

Figure3.15 A gaussian distribution. The probability of finding a value of x is given by the gaussian function f(x). The mean value of x is x_0, and the total width of the curve at half its maximum value is 2.35σ, whereσis the standard deviation of the distribution. The total probability of finding a value of x within a standard deviation of x_0 is equal to the shaded area and is 68.3 percent.

*Gaussian function: f(x)= $\frac{1}{\sigma\sqrt{2\pi}} e^{-(x-x_o)^2/2\sigma^2}$

Standard deviation $\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - x_o)^2}$ (square-root-mean)

Width of a gaussian curve at half its max is 2.35σ

$$p_{x_o \pm \sigma} = \int_{x_o-\sigma}^{x_o+\sigma} f(x)dx = 0.683$$

- Min ΔxΔk occur for Gaussian function

Take Δx,Δk as standard deviation ofφ(x)& g(x) ⟶ ΔxΔk=1/2 ∴ in general ΔxΔk≥ 1/2

∵ k=2π/λ = 2πP/h ⟶ P=hk/2π ΔP=hΔk/2π

$\because \Delta x \Delta k \geq 1/2 \qquad \Delta k \geq 1/2\Delta x$

$\longrightarrow \Delta x \Delta p \geq h/4\pi$ ($\because \Delta x(h\Delta k/2\pi) \geq h/4\pi$)

$\longrightarrow \Delta x \Delta p \geq \frac{\hbar}{2}$ [$\hbar = h/2\pi$]

Ex 3.6

A measurement establishes the position of a proton with an accuracy of $\pm$ $1.00\text{x}10^{-11}$m. Find the uncertainty in the proton's position 1.00s later. Assume v<<c

Sol: At time t=o, uncertainty in position$\Delta x_o = 1.00\text{x}10^{-11}$m

$\longrightarrow$ The uncertainty in P at this time $\geq \frac{\hbar}{2\Delta x_o}$

$\because \Delta P = m_o \Delta v \longrightarrow \Delta v = \Delta P/m_o \geq \frac{\hbar}{2m_o\Delta x_o}$

$\Delta x = t\Delta v \geq \frac{\hbar t}{2m_o\Delta x_o} = 3.15\text{x}10^{3}$m ($\because \Delta x \ \alpha \ 1/\Delta x_o$)

*the more we know at t=0, the less we know at t=t *

Figure3.16 An electron cannot be observed without changing its momentum.

look at e' light of wavelengthλ $\longrightarrow$ $P=h/\lambda$ $\longrightarrow$ when one of three photons bounces off the e' $\longrightarrow$ e' momentum is changed.

The exact P cannot be predicted, but $\Delta P \sim h/\lambda$ (the order of magnitude as P) $\Delta x \sim \lambda$

ie if we use shorterλ $\longrightarrow$ increase accuracy of position

$\longrightarrow$ higher photon momentum disturb e' motion more

$\longrightarrow$ accuracy of the momentum measurement deceasing

$\longrightarrow$ $\Delta x \Delta P \geqq h$ (consist with $\Delta x \Delta P \geqq \hbar/2$)

(1) If the energy is in the form of em waves, the limited time available restricts the accuracy with which we can determine the frequencyν.

(2) Assume the min uncertainty in the number of waves we count in a wave group is one wave.

$\because$ Frequency of wave = # of wave/time interval $\longrightarrow$ $\Delta\nu \geqq 1/\Delta t$

$\because$ $E=h\Delta\nu$ $\longrightarrow$ $\Delta E \geqq h/\Delta t$ or $\Delta E \Delta t \geqq h$

more precise calculation $\longrightarrow$ $\Delta E \Delta t \geqq \hbar/2$

ex 3.9

An "excited" atom gives up its excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom & the time it radiates is $1.0x10^{-9}$s. find the uncertainty in the frequency of the photon.

$$\Delta E \geqq \hbar/2\Delta t=5.3x10^{-27}J$$

$$\Delta \nu=\Delta E/h = 8x10^{6}Hz$$